Math 45 4.4 Word Problems with Linear Systems Ms. Carey

For each problem, write a linear system (or a single algebraic equation) and solve.

1) The sum of two numbers is 45. Twice the first number minus the second is 27. Find the numbers.

2) Fence on a lakefront property takes 240 feet for three sides, not including the lake. The width of the lot is 30 feet more than the length, and the long side is on the lake. What are the dimensions of the property?

3) Find the measures of two complementary angles such that the measure of the larger angle is 6° greater than twice the measure of the smaller angle. Find the angles.

4) A small aircraft flying west 500 miles takes 5 hours against the wind. Returning east with the wind, the trip takes 4 hours. Find the airspeed of the plane and the speed of the wind.

5) Rafael and Edith are running a marathon for charity. Rafael runs 12 mph and Edith 10 mph. Rafael lost his car keys and went back while Edith started on time. Rafael started 15 minutes late. How long will it take for him to catch up to her?

6) Two trains leave Chicago going opposite directions, one east, one west. The eastbound train goes 12 mph slower than the westbound. After 4 hours, they are 528 miles apart. Find the speed of each train.

Math 45 SSM 2/e 4.4 Word Problems with Linear Systems

Objectives

1) Direct translation that results in 2 equations and 2 unknown variables.
2) Geometry that results in 2 equations and 2 unknown variables
3) Uniform motion that results in 2 equations and 2 unknown variables.

Write a linear system and then solve the system using any valid method.

Valid Methods for Solving Systems

1) Graphing (not recommended)
2) Substitution
3) Elimination.

(1) The sum of two numbers is 45. Twice the first number minus the second is 27. Find the numbers.

$\boxed{x} + \boxed{y} = 45$ ← "sum of 2 numbers is 45."

twice the first minus the second is 27

$2 \cdot x - y = 27$

System

$$\begin{cases} x+y=45 & (A) \\ 2x-y=27 & (B) \end{cases}$$

solve by either method.

Elimination:

$$\frac{3x}{3} = \frac{72}{3}$$

$$x = 24$$

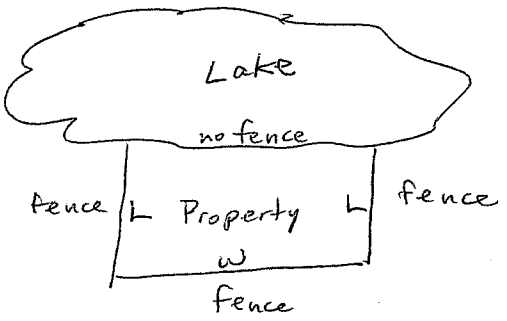
$x + y = 45 \longrightarrow 24 + y = 45 \longrightarrow y = 21$

$2x - y = 27 \longrightarrow 2(24) - y = 27 \longrightarrow 48 - 27 = y \longrightarrow y = 21$

answer $\boxed{24, 21}$

(2) Fence on a lakefront property takes 240 ft for 3 sides, not including the lake. Width of lot is 30 ft more than length. Long side is on lake. What are the dimensions of the property?

Draw diagram



"width is 30 feet more than length." means width is the longer side. Weird but true.

Fence: $L+L+W=240$ ← sum of 3 sides = 240 feet of fence

$2L+W=240$ combine like terms.

Direct translate: "width is 30 feet more than length"

$W = 30 + L$.

system: $\begin{cases} 2L+W=240 & (A) \\ W=30+L & (B) \end{cases}$

Solve by either method.

Substitution:

$2L+(30+L)=240$

$3L+30=240$

$-30 \quad -30$

$\frac{3L}{3} = \frac{210}{3}$

$\boxed{L=70 \text{ ft}}$

$W=30+L \Rightarrow W=30+70$

$\boxed{W=100 \text{ ft}}$

(3) Find the measures of two complementary angles such that the measure of the larger angle is 6° greater than twice the measure of the smaller angle.

$\boxed{x} + \boxed{y} = 90$ complementary angles

call x the larger angle: directly translate.

$$x = 6 + 2y$$

system: $\begin{cases} x + y = 90 & (A) \\ x = 6 + 2y & (B) \end{cases}$

Solve by either method.

substitution.

$$(6+2y) + y = 90$$
$$3y + 6 = 90$$
$$-6 \quad -6$$
$$\frac{3y}{3} = \frac{84}{3}$$
$$\boxed{y = 28^\circ}$$

Find x by $90 - y \Rightarrow 90 - 28 = 62^\circ$

or $x = 6 + 2y \Rightarrow 6 + 2(28) = 62^\circ$.

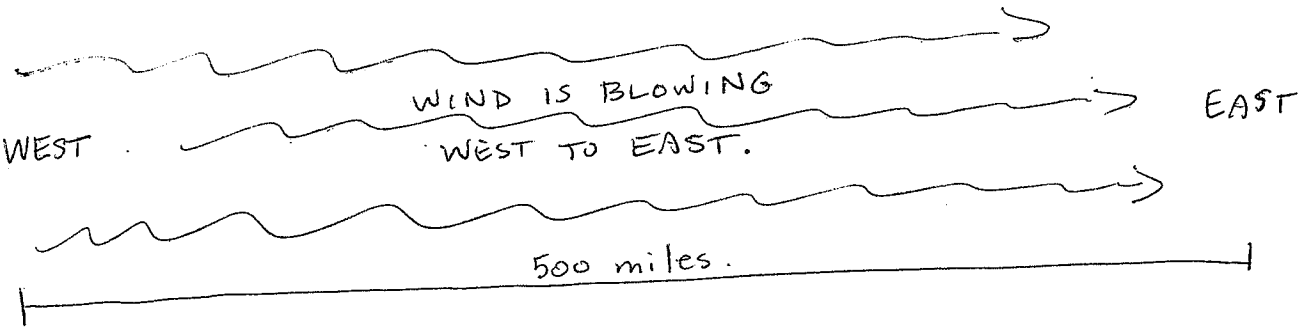
$$\boxed{x = 62^\circ}$$

Note: Some of these problems may be easier to solve using one equation and one variable. On a PQ or exam, you may use the set up you prefer, so long as it uses a equation(s) with variable(s).

One variable version of (3)

$\boxed{x} + \boxed{6+2x} = 90$

(4) A small aircraft flying west 500 miles takes 5 hrs against the wind; returning east with the wind, the trip takes 4 hours. Find the airspeed of the plane and the effect of the wind.



← plane flying west goes against wind
wind slows down the plane. (5 hrs)

R = plane speed − wind speed

$P - W$

plane flying east →
goes with wind.
wind speeds up plane (4 hrs)

R = plane speed + wind speed

$P + W$.

Chart:

	D	$=$ R	$\cdot$ T
West	500	$P-W$	5
East	500	$P+W$	4

Equations: $500 = (P-W)\cdot 5$ (A)
$500 = (P+W)\cdot 4$ (B)

Either dist or divide. I choose divide

$\frac{500}{5} = \frac{(P-W)\cdot 5}{5}$ (A) $\qquad \frac{500}{4} = \frac{(P+W)\cdot 4}{4}$ (B)

$100 = P-W \qquad 125 = P+W$

Solve by elimination

$P+W = 125$ (B)
$P-W = 100$ (A)

$2P = 225$

$P = 112.5$ mph plane

$112.5 - W = 100$

$W = 12.5$ mph wind

(5) Rafael and Edith are running a marathon for charity. Rafael runs 12 mph and Edith 10 mph. Rafael lost his car keys and went back while Edith started on time. Rafael started 15 minutes late. How long will it take for him to catch up to her?

$D = R \cdot T$

	D	R	T
Rafael	D	12	T
Edith.	D	10	T+.25

NOTE
* Time is in hours!
15 min = $\frac{1}{4}$ hr = .25 hr

$$\begin{cases} D = 12T & (A) \\ D = 10(T+.25) & (B) \end{cases}$$

solve by substitution (A) → (B)

$$12T = 10(T+.25)$$

dist $12T = 10T + 2.5$

$-10T \quad -10T$

$$\frac{2T}{2} = \frac{2.5}{2}$$

$T = 1.25$ hrs
$T = 1\frac{1}{4}$ hrs
$T = 1$ hr 15 min
$T = 75$ min

~~NOT 1 hr 25 min~~

The one variable version from chapter 2

$D = R \cdot T$

	R	T
12T	12	T
10(T+.25)	10	T+.25

Set distances equal

$$12T = 10(T+.25)$$

results in same equation as substitution method.

(6) Two trains leave Chicago going opposite directions, one east, one west. The east-bound train goes 12 mph slower than the west-bound. After 4 hours, they are 528 miles apart. Find the speed of each train.

	D = R · T		
east	4x	x	4
west	4y	y	4
Total	528		

x = east-bound train's rate

y = west-bound train's rate

$$\frac{4x}{4} + \frac{4y}{4} = \frac{528}{4}$$

$$\begin{cases} x + y = 132 & (A) \\ x = y - 12 & (B) \end{cases}$$

Directly translate

Substitution (B) → (A)

$$y - 12 + y = 132$$

$$2y - 12 = 132$$
$$+12 \quad +12$$

$$\frac{2y}{2} = \frac{144}{2}$$

$y = 72$ mph west-bound

$x = 72 - 12 =$ 60 mph east-bound

The one-variable version from chapter 2:

	D = R · T		
east	4(y−12)	y−12	4
west	4y	y	4
total	528		

opposite directions ⇒ add distances

$$4(y-12) + 4y = 528$$
$$4y - 48 + 4y = 528$$
$$8y - 48 = 528$$
$$8y = 576$$
$$y = 72 \text{ mph}$$ east

$y - 12 =$ (60 mph) west